HOLIDAY HOME WORK FOR SUMMER VACATION
CLASS – XII (MATHEMATICS)
ASSIGNMENTS
RELATION AND FUNCTIONS (20 Questions)

1. If A = {1, 2, 3, 4, 5}, write the relation a R b such that a + b = 8, a ,b € A. Write the domain, range & co-domain.
2. Define a relation R on the set **N** of natural numbers by R={(*x*,*y*):*y*= *x*+7, *x* is a natural number less than 4; *x*, *y* ∈ **N**}.Write down the domain and the range.
3. Let R be the relation in the set N given by R = {(a,b)| a = b – 2 , b > 6}. Check whether the relation is reflexive or not? Justify your answer.
4. Show that the relation R in the set N given by R = {(a , b)| a is divisible by b , a , b ∈ N} is reflexive and transitive but not symmetric.
5. Let R be the relation in the set N given by R = {(a ,b)| a > b} Show that the relation is neither reflexive nor symmetric but transitive.
6. Let R be the relation on R defined as (a , b) ∈ R iff 1+ ab > 0 ∀ a,b ∈R.Show that R is reflexive, symmetric but not transitive.
7. Check whether the relation R is reflexive, symmetric and transitive.
   R = { (x ,y)| x – 3y = 0} on A ={1,2,3……….13,14}.
8. If f(x) = $x^2 - x^{-2}$, then find f(1/x).
9. Show that the function f: N→N given by f(x) = 2x is one-one but not onto.
10. Show that the signum function f: R→R given by: $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$

    is neither one-one nor onto.
11. Let A = {-1, 0, 1} and B = {0 , 1}. State whether the function f : A → B defined by f(x) = x^2 is bijective .
12. Let f(x) = $\frac{x-1}{x+1}$, x ≠ -1,then find $f^{-1}(x)$
13. If f(x) = e^{2x} and g(x) = log √x , x > 0, find
    (a) (f + g)(x) (b) (f .g)(x) (c) f o g (x) (d) g o f (x).
14. If f(x) = $\frac{x-1}{x+1}$, then show that (a) $f\left(\frac{1}{x}\right) = -f(x)$ (b) $f\left(-\frac{1}{x}\right) = \frac{-1}{f(x)}$
15. Let * be the binary operation on N given by a*b = LCM of a &b . Find 3*5.
16. Let *be the binary on N given by a*b =HCF of a ,b , a,b∈N. Find 20*16.
17. Let * be a binary operation on the set Q of rational numbers defined as $a * b = \frac{ab}{5}$.

    Write the identity of *, if any.

18. If a binary operation '*' on the set of integer Z , is defined by a * b = a + $3b^2$,then find the value of 2 * 4.

19. Let A = {1,2,3}, B = {4,5,6,7} and let f = {(1 , 4),(2 , 5), (3 , 6)} be a function from A to B. State whether f is one-one or not.

20. If f: R→R is defined by f(x) = $\frac{3x+5}{2}$. Find f^{-1}

INVERSE TRIGONOMETRIC FUNCTIONS (20 Questions)

Write the principal value of the following :

1. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

2. $\sin^{-1}\left(-\frac{1}{2}\right)$

3. $\tan^{-1}\left(-\sqrt{3}\right)$

4. $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

5 . Evaluate $\cot[\tan^{-1} a + \cot^{-1} a]$

6. Find x if $sec^{-1}(\sqrt{2}) + cosec^{-1}x = \frac{\pi}{2}$

7 . Evaluate $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

8 . Evaluate $Sin^{-1}\left(sin\frac{4\pi}{5}\right)$

9. Evaluate $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

10. Evaluate $tan^{-1}\left(tan\frac{7\pi}{6}\right)$

11. **Find the principal value of $sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$**

12 Evaluate $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

13 Express in simplest form: $\sin^{-1}[3x - 4x^3]$

14 Evaluate: $\sin(\cot^{-1}x)$

15 For $\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$, find the value of θ

16 Evaluate: $tan^{-1}(\sqrt{3}) - sec^{-1}(-2)$

17 Express in simplest form: $tan^{-1}\left[\frac{\cos x}{1-\sin x}\right]$

18 Evaluate: $sin\left[\frac{\pi}{3} - sin^{-1}\left(-\frac{1}{2}\right)\right]$

19 Find the Principal value of $sin^{-1}\left(sin\frac{2\pi}{3}\right) + tan^{-1}\left(tan\frac{7\pi}{6}\right)$

20 Evaluate: $sec^{-1}\left[sec\left(-\frac{\pi}{4}\right)\right]$

MATRICES (20 QUESTIONS)

1. If a matrix has 5 elements, what are the possible orders it can have?
2. Construct a 3 × 2 matrix whose elements are given by $a_{ij} = \frac{1}{2}|i - 3j|$
3. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, then find A –2 B.
4. If $A = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$, write the order of AB and BA.
5. Find the co-factor of a_{12} in $A = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$
6. For the following matrices A and B, verify $(AB)^T = B^T A^T$,

   where $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$
7. Give example of matrices A & B such that AB = O, but BA ≠ O, where O is a zero matrix and A, B are both non zero matrices.
8. If B is skew symmetric matrix, write whether the matrix (ABA^T) is symmetric or skew symmetric.
9. $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find a and b so that $A^2 + aI = bA$
10. Find the co-factor of a_{23} in $A = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$
11. Find the adjoint of the matrix $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$
12. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, write A^{-1} .
13. Verify A(adjA) = (adjA) A = $|A|$I if $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$
14. Verify A(adjA) = (adjA) A = $|A|$I if $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$
15. If A is square matrix satisfying $A^2 = I$, then what is the inverse of A ?
16. For what value of k , the matrix $A = \begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$ is not invertible ?
17. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$. Show that $A^2 - 5A - 14I = 0$. Hence find A^{-1}
18. Evaluate $\begin{vmatrix} cos15^\circ & sin15^\circ \\ sin75^\circ & cos75^\circ \end{vmatrix}$

19. What is the value of $|3I|$, where I is identity matrix of order 3?
20. If A is non singular matrix of order 3 and $|A|$ = 3, then find $|2A|$

DETERMINANTS (20 questions)

1 If A = $\begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations:
2x – 3y + 5z = 11, 3x + 2y – 4z = - 5, x + y – 2z = - 3

2. Using matrices, solve the following system of equations:
   a. x + 2y - 3z = - 4
      2x + 3y + 2z = 2
      3x - 3y – 4z = 11

   b. 4x + 3y + 2z = 60
      x + 2y + 3z = 45
      6x + 2y + 3z = 70

3. Find the product AB, where A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and use it to solve the equations x – y = 3, 2x + 3y + 4z = 17, y + 2z = 7
4. Using matrices, solve the following system of equations:
   $\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 4$
   $\frac{2}{x} + \frac{1}{y} - \frac{3}{z} = 0$
   $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2$
5. Using elementary transformations, find the inverse of the matrix
   $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

6 Using properties of determinants, solve the following for x :
$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

7 Using properties of determinants, solve the following for x :
$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

8 Using properties of determinants, solve the following for x :
$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

9 If a, b, c, are positive and unequal, show that the following determinant is negative:

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

10. prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

11. prove that $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc$

12. prove that $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$

13. prove that $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$

14 prove that $\begin{vmatrix} (b+c)2 & ab & ca \\ ab & (a+c)2 & bc \\ ac & bc & (a+b)2 \end{vmatrix} = 2abc(a+b+c)^3$

15 If p, q, r are not in G.P and $\begin{vmatrix} 1 & \frac{q}{p} & \alpha+\frac{q}{p} \\ 1 & \frac{r}{q} & \alpha+\frac{r}{q} \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$, show that $p\alpha^2+2p\alpha+r=0$.

16. If a, b, c are real numbers, and $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$

Show that either a + b +c = 0 or a = b = c.

17. Using properties of determinants, prove that : $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

18. Using properties of determinants, prove that :

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

19. Using properties of determinants, prove that : $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

20. Express $A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

21 Let $A = \begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix}$, prove by mathematical induction that : $A^n = \begin{bmatrix} 1-2n & -4n \\ n & 1+2n \end{bmatrix}$.

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